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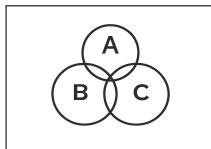
Discrete Distribution Standard Deviation

M



$$X = \{1, 2, 3, 4, 5, 6\}$$

$$P(X = x) = \frac{1}{6}$$



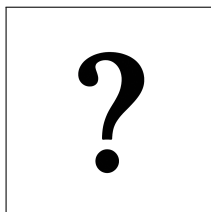
$$E(X) = \sum_{x=1}^6 x \cdot \frac{1}{6} = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5$$

$$E(X^2) = \sum_{x=1}^6 x^2 \cdot \frac{1}{6} = \frac{1^2+2^2+3^2+4^2+5^2+6^2}{6} = \frac{91}{6}$$

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = \frac{91}{6} - (3.5)^2 = \frac{91}{6} - \frac{49}{4} = \frac{182-147}{12} = \frac{35}{12} = \frac{35}{12}$$

$$\text{SD}(X) = \sqrt{\text{Var}(X)} = \sqrt{\frac{35}{12}} = \sqrt{\frac{35}{12}} \approx 1.708$$

$$\text{Var}(4X + 2) = 16 \text{Var}(X)$$



$$P(0 < X \leq 20) = 0.06 : P(20 < X \leq 30) = 0.18$$

$$P(30 < X \leq 40) = 0.26 : P(40 < X \leq 50) = 0.22$$

$$P(50 < X \leq 60) = 0.14 : P(60 < X \leq 70) = 0.06$$

$$P(70 < X \leq 80) = 0.04 : P(80 < X \leq 90) = 0.04$$

$$P(X > 90) = 0.00$$

$$Y(X < 50) = 0 \rightarrow P(Y = 0) = 0.72$$

$$Y(50 \leq X < 60) = 300 \rightarrow P(Y = 300) = 0.14$$

$$Y(60 \leq X < 70) = 600 \rightarrow P(Y = 600) = 0.06$$

$$Y(X \geq 70) = 700 \rightarrow P(Y = 700) = 0.08$$

$$E(Y) = ? : E(Y^2) = ? : \text{Var}(Y) = ? : \sigma_Y = ?$$

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$$\begin{aligned} P(0 < X \leq 20) &= 0.06 : P(20 < X \leq 30) = 0.18 \\ P(30 < X \leq 40) &= 0.26 : P(40 < X \leq 50) = 0.22 \\ P(50 < X \leq 60) &= 0.14 : P(60 < X \leq 70) = 0.06 \\ P(70 < X \leq 80) &= 0.04 : P(80 < X \leq 90) = 0.04 \\ P(X > 90) &= 0.00 \end{aligned}$$

$$\begin{aligned} Y(X < 50) &= 0 \rightarrow P(Y = 0) = 0.72 \\ Y(50 \leq X < 60) &= 300 \rightarrow P(Y = 300) = 0.14 \\ Y(60 \leq X < 70) &= 600 \rightarrow P(Y = 600) = 0.06 \\ Y(X \geq 70) &= 700 \rightarrow P(Y = 700) = 0.08 \end{aligned}$$

$$E(Y) = ? : E(Y^2) = ? : Var(Y) = ? : \sigma_Y = ?$$

$$\begin{aligned} E(Y) &= 0 \cdot 0.72 + 300 \cdot 0.14 + 600 \cdot 0.06 + 700 \cdot 0.08 \\ E(Y) &= 42 + 36 + 56 \\ E(Y) &= 134 \end{aligned}$$

$$\begin{aligned} E(Y^2) &= 0^2 \cdot 0.72 + 300^2 \cdot 0.14 + 600^2 \cdot 0.06 + 700^2 \cdot 0.08 \\ E(Y^2) &= 90,000 \cdot 0.14 + 360,000 \cdot 0.06 + 490,000 \cdot 0.08 \\ E(Y^2) &= 12,600 + 21,600 + 39,200 \\ E(Y^2) &= 73,400 \end{aligned}$$

$$\begin{aligned} Var(Y) &= E(Y^2) - [E(Y)]^2 \\ Var(Y) &= 73,400 - (134)^2 \\ Var(Y) &= 73,400 - 17,956 \\ Var(Y) &= 55,444. \end{aligned}$$

$$\begin{aligned} \sigma_Y &= \sqrt{Var(Y)} \\ \sigma_Y &= \sqrt{55,444} \\ \sigma_Y &\approx 235.5 \end{aligned}$$