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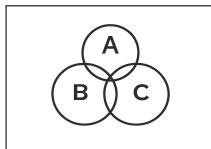
Discrete Distribution Standard Deviation

C



$$X = \{1, 2, 3, 4, 5, 6\}$$

$$P(X = x) = \frac{1}{6}$$



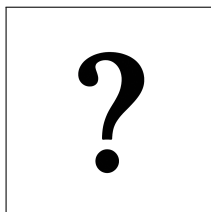
$$E(X) = \sum_{x=1}^6 x \cdot \frac{1}{6} = \frac{1+2+3+4+5+6}{6} = \frac{21}{6} = 3.5$$

$$E(X^2) = \sum_{x=1}^6 x^2 \cdot \frac{1}{6} = \frac{1^2+2^2+3^2+4^2+5^2+6^2}{6} = \frac{91}{6}$$

$$\begin{aligned} \text{Var}(X) &= E(X^2) - [E(X)]^2 = \frac{91}{6} - (3.5)^2 = \frac{91}{6} - \frac{49}{4} = \\ &= \frac{182-147}{12} = \frac{35}{12} = \frac{35}{12} \end{aligned}$$

$$\text{SD}(X) = \sqrt{\text{Var}(X)} = \sqrt{\frac{35}{12}} = \sqrt{\frac{35}{12}} \approx 1.708$$

$$\text{Var}(4X + 2) = 16 \text{Var}(X)$$



$$\mu = E[X] = ?$$

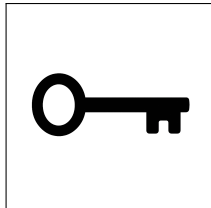
$$E[X^2] = ?$$

$$\text{Var}(X) = ?$$

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Discrete Distribution Standard Deviation

C



$$x = \{-2, -1, 0, 1, 2\}$$

$$p(x) = 1/55 (x - 3)^2$$

$$P(X = -2) = 25 \times 1/55 = 25/55$$

$$P(X = -1) = 16 \times 1/55 = 16/55$$

$$P(X = 0) = 9 \times 1/55 = 9/55$$

$$P(X = 1) = 4 \times 1/55 = 4/55$$

$$P(X = 2) = 1 \times 1/55 = 1/55$$

$$\mu = E[X] = ?$$

$$E[X^2] = ?$$

$$\text{Var}(X) = ?$$

$$-2 \cdot 25 = -50$$

$$-1 \cdot 16 = -16$$

$$0 \cdot 9 = 0$$

$$1 \cdot 4 = 4$$

$$2 \cdot 1 = 2$$

$$\sum x(x-3)^2 = -50 - 16 + 0 + 4 + 2 = -60$$

$$\mu = \sum_{x=-2}^2 x p(x) = \frac{1}{55} \sum_{x=-2}^2 x(x-3)^2 = \frac{-60}{55} = -\frac{12}{11} \approx -1.0909$$

$$E[X^2] = \sum_{x=-2}^2 x^2 p(x) = \frac{1}{55} \sum_{x=-2}^2 x^2 (x-3)^2$$

$$E[X^2] = \sum x^2 (x-3)^2 = 4 \cdot 25 + 1 \cdot 16 + 0 \cdot 9 + 1 \cdot 4 + 4 \cdot 1$$

$$E[X^2] = \sum x^2 (x-3)^2 = 100 + 16 + 0 + 4 + 4 = 124$$

$$E[X^2] = \frac{124}{55} \approx 2.2545$$

$$\text{Var}(X) = E[X^2] - \mu^2$$

$$\text{Var}(X) = \frac{124}{55} - \left(-\frac{12}{11}\right)^2$$

$$\text{Var}(X) = \frac{124}{55} - \frac{144}{121}$$

$$\text{Var}(X) = \frac{1364 - 720}{605}$$

$$\text{Var}(X) = \frac{644}{605}$$

$$\text{Var}(X) \approx 1.0645$$